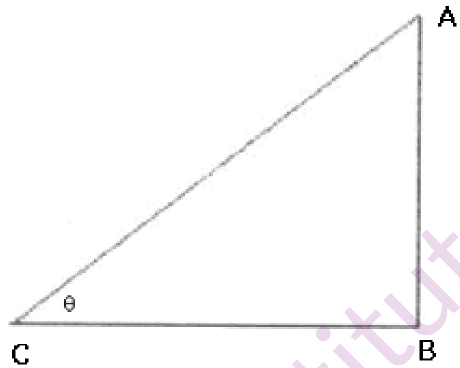
CBSE Board
Class X Mathematics
Board Paper Solution – 2012

Time: 3 hours | Total Marks: 90

Section A

1. Correct answer: B



Let AB be the tower and BC be its shadow. Let θ be the angle of elevation of the sun.

According to the given information,

$BC = \sqrt{3}\,AB$ ... (1)

In ΔABC,

$\tan\theta = \frac{AB}{BC} = \frac{AB}{\sqrt{3}AB} = \frac{1}{\sqrt{3}}$ [Using (1)]

We know that $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$

Hence, the angle of elevation of the sun is 30°.

www.akilaninstitute.com

2. Correct answer: B

Diameters of two circles are given as 10 cm and 24 cm.

Radius of one circle = r_1 = 5 cm, Radius of other circle = r_2 = 12 cm

According to the given information,

Area of the larger circle = $\pi(r_1)^2 + \pi(r_2)^2$

$$= \pi(5)^2 + \pi(12)^2$$
$$= \pi(25 + 144)$$
$$= 169\pi$$
$$= \pi(13)^2$$

$\therefore$Radius of larger circle = 13 cm

Hence, the diameter of larger circle = 26 cm

3. Correct answer: C

Let the original radius and the height of the cylinder be r and h respectively.

Volume of the original cylinder = $\pi r^2 h$

Radius of the new cylinder = $\frac{r}{2}$

Height of the new cylinder = h

Volume of the new cylinder = $\pi\left(\frac{r}{2}\right)^2 h = \frac{\pi r^2 h}{4}$

$$\text{Required ratio} = \frac{\text{Volume of the new cylinder}}{\text{Volume of the original cylinder}} = \frac{\frac{\pi r^2 h}{4}}{\pi r^2 h} = \frac{1}{4} = 1:4$$

4. Correct answer: C

When two dice are thrown together, the total number of outcomes is 36.

Favourable outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$$\therefore \text{Required probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{6}{36} = \frac{1}{6}$$

www.aklanininstitute.com

5. Correct answer: B

It is given that the point P divides AB in the ratio 2: 1.

Using section formula, the coordinates of the point P are

$$\left(\frac{1\times 1+2\times 4}{2+1},\frac{1\times 3+2\times 6}{2+1}\right)=\left(\frac{1+8}{3},\frac{3+12}{3}\right)=(3,5)$$

Hence the coordinates of the point P are (3, 5).

6. Correct answer: A

Let the coordinates of the other end of the diameter be (x, y).

We know that the centre is the mid-point of the diameter. So, O(-2, 5) is the mid-point of the diameter AB. The coordinates of the point A and B are (2, 3) and (x, y) respectively.

Using mid-point formula, we have,

$$-2=\frac{2+x}{2}\Rightarrow -4=2+x\Rightarrow x=-6$$

$$5=\frac{3+y}{2}\Rightarrow 10=3+y\Rightarrow y=7$$

Hence, the coordinates of the other end of the diameter are (-6, 7).

www.akhaninstitute.com

7. Correct answer: C

The first 20 odd numbers are 1, 3, 5, … … 39

This is an AP with first term 1 and the common difference 2.

Sum of 20 terms = S_{20}

$$S_{20}=\frac{20}{2}\left[2(1)+(20-1)(2)\right]=10\left[2+38\right]=400$$

Thus, the sum of first 20 odd natural numbers is 400.

8. Correct answer: A

It is given that 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$.

Therefore, $y = 1$ will satisfy both the equations.

$\therefore a(1)^2 + a(1) + 3 = 0$

$\Rightarrow a + a + 3 = 0$

$\Rightarrow 2a + 3 = 0$

$\Rightarrow a = \frac{-3}{2}$

Also, $(1)^2 + (1) + b = 0$

$\Rightarrow 1 + 1 + b = 0 \Rightarrow b = -2$

$\therefore ab = \frac{-3}{2} \times -2 = 3$

9. Correct answer: B

It is known that the lengths of tangents drawn from a point outside a circle are equal in length.

Therefore, we have:

AP = AR ... (1) (Tangents drawn from point A)

BP = BQ ... (2) (Tangents drawn from point B)

CQ = CR ... (3) (Tangents drawn from point C)

Using the above equations,

AR = 4 cm (AP = 4 cm, given)

BQ = 3 cm (BP = 3 cm, given)

AC = 11 cm $\Rightarrow$ RC = 11 cm – 4 cm = 7 cm

$\Rightarrow$ CQ = 7 cm

Hence, BC = BQ + CQ = 3 cm + 7 cm = 10 cm

www.akilaninstitute.com

10. Correct answer: A

It is known that the tangents from an external point to the circle are equal.

$\therefore$EK = EM, DK = DH and FM = FH ... (1)

Perimeter of ΔEDF = ED + DF + FE

= (EK − DK) + (DH + HF) + (EM − FM)

= (EK − DH) + (DH + HF) + (EM − FH) [Using (1)]

= EK + EM

= 2 EK = 2 (9 cm) = 18 cm

Hence, the perimeter of ΔEDF is 18 cm.

SECTION B

11. It is given that the point A (0, 2) is equidistant from the points B(3, p) and C(p, 5).

So, AB = AC $\Rightarrow$ $AB^2 = AC^2$

Using distance formula, we have

$\Rightarrow (0-3)^2 + (2-p)^2 = (0-p)^2 + (2-5)^2$
$\Rightarrow 9 + 4 + p^2 - 4p = p^2 + 9$
$\Rightarrow 4 - 4p = 0$
$\Rightarrow 4p = 4$
$\Rightarrow p = 1$

Hence, the value of p = 1.

www.akilaninstitute.com

12. Total number of outcomes is 50.

Favourable outcomes = {12, 24, 36, 48}

$\therefore$Required probability $= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{4}{50} = \dfrac{2}{25}$

13.

Given volume of a hemisphere = $2425\frac{1}{2}\text{ cm}^3 = \frac{4851}{2}\text{ cm}^3$

Now, let r be the radius of the hemisphere

Volume of a hemisphere = $\frac{2}{3}\pi r^3$

$\therefore \quad \frac{2}{3}\pi r^3 = \frac{4851}{2}$

$\Rightarrow \quad \frac{2}{3}\times\frac{22}{7}\times r^3 = \frac{4851}{2}$

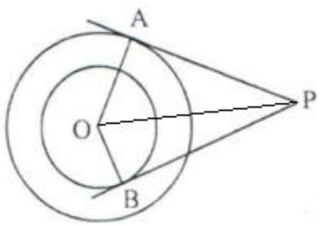
$\Rightarrow \quad r^3 = \frac{4851}{2}\times\frac{3}{2}\times\frac{7}{22} = \left(\frac{21}{2}\right)^3$

$\therefore \quad r = \frac{21}{2}\text{ cm}$

So, Curved surface area of the hemisphere = $2\pi r^2$

$$= \cancel{2} \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{21}^{3}}{\cancel{2}} \times \frac{21}{\cancel{2}} = 693 \text{ sq.cm}$$

14. Given: Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii OA = 8 cm, OB = 5 cm respectively. Also, AP = 15 cm



Construction: We join the points O and P.

Solution: $OA \perp AP$; $OB \perp BP$

[Using the property that radius is perpendicular to the tangent at the point of contact of a circle]

In right angled triangle OAP,

$OP^2 = OA^2 + AP^2$ [Using Pythagoras Theorem]

$= (8)^2 + (15)^2 = 64 + 225 = 289$

$\therefore$ OP = 17 cm

In right angled triangle OBP,

$OP^2 = OB^2 + BP^2$

$\Rightarrow BP^2 = OP^2 - OB^2 = (17)^2 - (5)^2 = 289 - 25 = 264$

$\therefore \quad BP = \sqrt{264} = 2\sqrt{66}\text{ cm}$

www.aklianinstitute.com

15. Given: ABC is an isosceles triangle, where AB = AC, circumscribing a circle.

To prove: The point of contact P bisects the base BC. i.e. BP = PC

Proof: It can be observed that

BP and BR; CP and CQ; AR and AQ are pairs of tangents drawn to the circle from the external points B, C and A respectively.

Since the tangents drawn from an external point to a circle, then

BP = BR --- (i)

CP = CQ --- (ii)

AR = AQ --- (iii)

Given that AB = AC

⇒ AR + BR = AQ + CQ

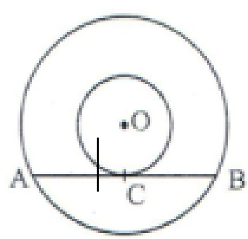
⇒ BR = CQ [from (iii)]

⇒ BP = CP [from (i) and (ii)]

∴ P bisects BC.

OR

Given: The chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C.



To prove: AC = CB

Construction: Let us join OC.

Proof: In the smaller circle, AB is a tangent to the circle at the point of contact C.

∴ OC ⊥ AB ------ (i)

(Using the property that the radius of a circle is perpendicular to the tangent at the point of contact)

For the larger circle, AB is a chord and from (i) we have OC ⊥ AB

∴ OC bisects AB

(Using the property that the perpendicular drawn from the centre to a chord of a circle bisects the chord)

∴ AC = CB

www.akilaninstitute.com

16. Given, OABC is a square of side 7 cm

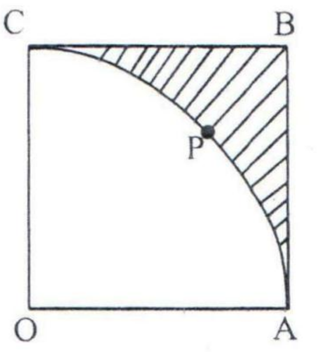
i.e. OA = AB = BC = OC = 7cm

∴ Area of square OABC = $(\text{side})^2 = 7^2 = 49$ sq.cm

Given, OAPC is a quadrant of a circle with centre O.

∴ Radius of the sector = OA = OC = 7 cm.

Sector angle = 90^o



∴ Area of quadrant OAPC $= \frac{90^o}{360^o} \times \pi r^2$

$$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 = \frac{77}{2} \text{ sq.cm} = 38.5 \text{ sq.cm}$$

∴ Area of shaded portion = Area of Square OABC − Area of quadrant OAPC

$$= (49 - 38.5) \text{ sq. cm} = 10.5 \text{ sq.cm}$$

17. First three- digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore the series is 105, 112, 119,…

The maximum possible three digit number is 999.

When we divide by 7, the remainder will be 5.

Clearly, 999 – 5 = 994 is the maximum possible three – digit number divisible by 7.

The series is as follows:

105, 112, 119, …., 994

Here a = 105, d = 7

Let 994 be the nth term of this A.P.

www.akilaninstitute.com

$$a_n = a + (n-1)d$$

$\Rightarrow \quad 994 = 105 + (n-1)7$

$\Rightarrow \quad (n-1)7 = 889$

$\Rightarrow \quad (n-1) = 127$

$\Rightarrow \quad n = 128$

So, there are 128 terms in the A.P.

$$\therefore \quad \text{Sum} = \frac{n}{2}\{\text{first term} + \text{last term}\}$$

$$= \frac{128}{2}\{a_1 + a_{128}\}$$

$$= 64\{105 + 994\} = (64)(1099) = 70336$$

18. Given quadratic equation is $3x^2 - 2kx + 12 = 0$

Here a = 3, b = -2k and c = 12

The quadratic equation will have equal roots if Δ = 0

$\therefore \quad b^2 - 4ac = 0$

Putting the values of a, b and c we get

$$(-2k)^2 - 4(3)(12) = 0$$

$\Rightarrow \quad 4k^2 - 144 = 0$

$\Rightarrow \quad 4k^2 = 144$

$\Rightarrow \quad k^2 = \frac{144}{4} = 36$

Considering square root on both sides,

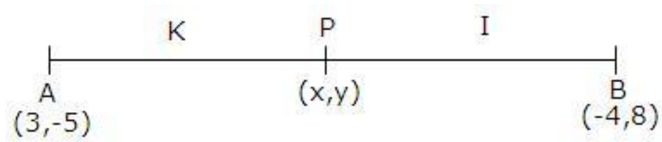
$$k = \sqrt{36} = \pm 6$$

Therefore, the required values of k are 6 and -6.

www.akilaninstitute.com

SECTION – C

19.



Let the co-ordinates of point P be (x, y)

Then using the section formula co-ordinates of P are.

$x = \frac{-4K+3}{K+1}$ $\qquad$ $y = \frac{8K-5}{K+1}$

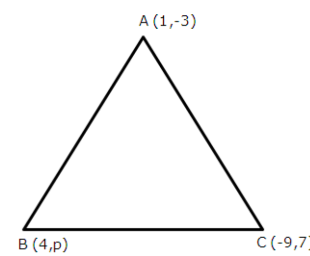
Since P lies on x+y=0

$\therefore \quad \frac{-4K+3}{K+1} + \frac{8K-5}{K+1} = 0$

$\Rightarrow 4K - 2 = 0 \Rightarrow k = \frac{2}{4} \Rightarrow K = \frac{1}{2}$

Hence the value of $K = \frac{1}{2}$.

20. The area of a triangle, whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is



$$\Delta = \frac{1}{2} | x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) |$$

Substituting the given coordinates

Area of $\Delta = \frac{1}{2} | 1(p-7) + 4(7+3) + (-9)(-3-p) |$

$\Rightarrow \frac{1}{2} |(p-7) + 40 + 27 + 9p| = 15$

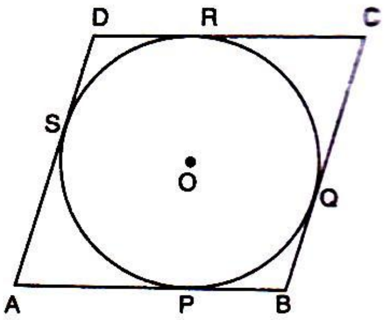
$\Rightarrow 10p + 60 = \pm 30$

$\Rightarrow 10p = -30 \quad$ or $\quad 10p = -90$

$\Rightarrow p = -3. \quad$ or $\quad p = -9$

www.akilaninstitute.com

21. Let ABCD be a parallelogram such that its sides touching a circle with centre O. We know that the tangents to a circle from an exterior point are equal in length.



$\therefore$ AP = AS [From A] ...(i)

BP = BQ [From B] ...(ii)

CR = CQ [From C] ...(iii)

and, DR = DS [From D] ...(iv)

Adding (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

$\Rightarrow$ 2AB = 2BC [$\because$ ABCD is a parallelogram $\therefore$ AB=CD and BC = AD]

$\Rightarrow$ AB=BC

Thus, AB = BC = CD = AD

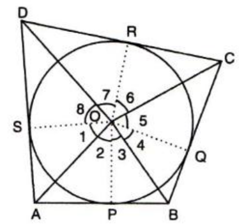
Hence, ABCD is a rhombus.

www.akilaninstitute.com

OR

A circle with centre O touches the sides AB, BC, CD, and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.

TO PROVE $\angle AOB + \angle COD = 180^o$ and, $\angle AOD + \angle BOC = 180^o$



CONSTRUCTION Join OP, OQ, OR and OS.

PROOF Since the two tangents drawn from a external point to a circle subtend equal angles at the centre.

$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6$ and $\angle 7 = \angle 8$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

$\left[\begin{array}{l}\text{Sum of all the angles} \\ \text{subtended at a point is } 360^o\end{array}\right]$

$\Rightarrow 2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^o$ and $2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^o$

$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^o$ and $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^o$

$\left[\begin{array}{l}\because \angle 2 + \angle 3 = \angle AOB, \angle 6 + \angle 7 = \angle COD \\ \angle 1 + \angle 8 = \angle AOD \text{ and } \angle 4 + \angle 5 = \angle BOC\end{array}\right]$

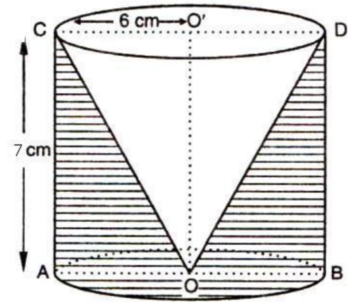
$\Rightarrow \angle AOB + \angle COD = 180^o$

and $\angle AOD + \angle BOC = 180^o$

Hence Proved

www.akilaninstitute.com

22. Given: radius of cyl=radius of cone=r=6cm



Height of the cylinder=height of the cone=h=7cm

Slant height of the cone= l

$\sqrt{7^2+6^2}=\sqrt{85}$ cm

Total surface area of the remaining solid

=curved surface area of the cylinder + area of the base of the cylinder + curved surface area of the cone

$$(2\pi rh+\pi r^2+\pi rl)$$
$$=2x\frac{22}{7}\times 6\times 7\;+\frac{22}{7}\times 6^2+\frac{22}{7}\times 6\times\sqrt{85}$$
$$=264+\frac{792}{7}+\frac{132}{7}\sqrt{85}$$
$$=377.1+\frac{132}{7}\sqrt{85}\text{ cm}^2$$

OR

Volume of the conical heap=volume of the sand emptied from the bucket.

Volume of the conical heap=

$\frac{1}{3}\pi r^2h=\frac{1}{3}\pi r^2 x24\,cm^2$ (height of the cone is 24)----------(1)

Volume of the sand in the bucket=πr^2h

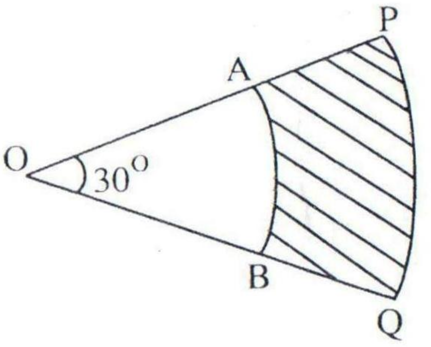
$$=\pi(18)^2 32cm^2 ------(2)$$

Equating 1 and 2

$$\frac{1}{3}\pi r^2x24=\pi(18)^2 32$$
$$\Rightarrow r^2=\frac{(18)^2x32x3}{24}$$
$$\Rightarrow r=36cm$$

www.akilaninstitute.com

23. Area of the shaded region= Area of sector POQ - Area of sector AOB



Area of Shaded region $= (\frac{\theta}{360}\pi R^2 - \frac{\theta}{360}\pi r^2)$

$$= \frac{30}{360} \times \frac{22}{7} \times (7^2 - 3.5^2)$$

$$= \frac{77}{8} cm^2$$

24.

$$4x^2 - 4ax + (a^2 - b^2) = 0$$

$$\Rightarrow (4x^2 - 4ax + a^2) - b^2 = 0$$

$$\Rightarrow \left[(2x)^2 - 2.2x.a + a^2\right] - b^2 = 0$$

$$\Rightarrow \left[(2x - a)^2\right] - b^2 = 0$$

$$\Rightarrow \left[(2x - a) - b\right]\left[(2x - a) + b\right] = 0$$

$$\Rightarrow \left[(2x - a) - b\right] = 0 \quad \text{or} \quad \left[(2x - a) + b\right] = 0$$

$$\Rightarrow x = \frac{a + b}{2}; x = \frac{a - b}{2}$$

OR

$$3x^2 - 2\sqrt{6}\,x + 2 = 0$$

$$\Rightarrow \quad 3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$$

$$\Rightarrow \quad \sqrt{3}\,x\left[\sqrt{3}\,x - \sqrt{2}\right] - \sqrt{2}\left[\sqrt{3}\,x - \sqrt{2}\right] = 0$$

$$\Rightarrow \quad \left(\sqrt{3}\,x - \sqrt{2}\right)\left(\sqrt{3}\,x - \sqrt{2}\right) = 0$$

$$\Rightarrow \quad \left(\sqrt{3}\,x - \sqrt{2}\right)^2 = 0$$

$$\therefore \quad \sqrt{3}\,x - \sqrt{2} = 0$$

$$\Rightarrow \quad \sqrt{3}\,x = \sqrt{2}$$

$$\Rightarrow \quad x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2} \times \sqrt{3}}{\left(\sqrt{3}\right)^2} = \frac{\sqrt{6}}{3}$$

www.akilaninstitute.com

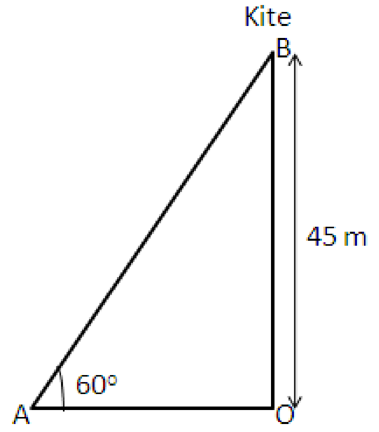
25. Given: Position of kite is B.

Height of kite above ground= 45 m

Angle of inclination = 60°

Required length of string = AB

In right angled triangle AOB,



$$\sin A = \frac{OB}{AB} \Rightarrow \sin 60^o = \frac{45}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{45}{AB}$$

$$\Rightarrow AB = \frac{45 \times 2}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 30\sqrt{3} \text{ m}$$

Hence, the length of the string is $30\sqrt{3}$ m

26. It is given that $\angle A = 105^\circ$, $\angle C = 30^\circ$.

Using angle sum property of triangle, we get, $\angle B = 45^\circ$

The steps of construction are as follows:

1. Draw a line segment BC = 6 cm.

2. At B, draw a ray making an angle of 45° with BC.

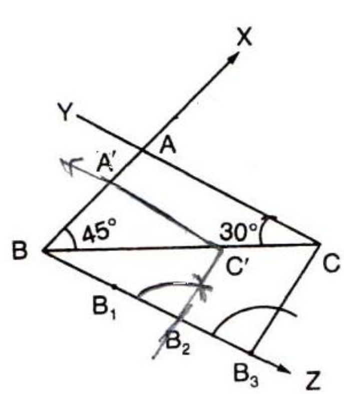
3. At C, draw a ray making an angle of 30° with BC. Let the two rays meet at point A.

4. Below BC, make an acute $\angle CBX$. Along BX mark off three points B_1, B_2, B_3, such that $BB_1 = B_1B_2 = B_2B_3$. Join B_3C.

7. From B2, draw B2C′ || B3C.

8. From C′, draw C′A′ || CA, meeting BA at the point A′.

Then A′BC′ is the required triangle.



www.akilaninstitute.com

27. Let a and d respectively be the first term and the common difference of the AP.

We know that the n^{th} term of an AP is given by $a_n = a + (n - 1)d$

According to the given information,

$a_{16} = 1 + 2\, a_8$

$\Rightarrow a + (16 - 1)d = 1 + 2[a + (8 - 1)d]$

$\Rightarrow a + 15d = 1 + 2a + 14d$

$\Rightarrow a + 15d = 1 + 2a + 14d$

$\Rightarrow -a + d = 1 \qquad \dots (1)$

Also, it is given that, $a_{12} = 47$

$\Rightarrow a + (12 - 1)d = 47$

$\Rightarrow a + 11d = 47 \qquad \dots (2)$

Adding (1) and (2), we have:

$12d = 48$

$\Rightarrow d = 4$

From (1),

$-a + 4 = 1 \Rightarrow a = 3$

Hence, $a_n = a + (n - 1)d = 3 + (n - 1)(4) = 3 + 4n - 4 = 4n - 1$

Hence, the n^{th} term of the AP is $4n - 1$.

www.akilaninstitute.com

28. Total number of outcomes = 52

(i) Probability of getting a red king

Here the number of favourable outcomes = 2

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{2}{52} = \frac{1}{26}$$

(ii) Probability of getting a face card

Total number of face cards = 12

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{12}{52} = \frac{3}{13}$$

(iii) Probability of queen of diamonds

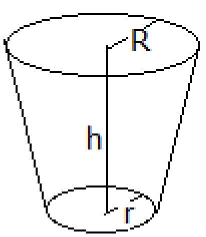
Number of queens of diamond = 1

$$\text{Probability} = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{52}$$

www.akilaninstitute.com

SECTION – D

29. Here, R = 28 cm and r = 21 cm, we need to find h.



Volume of frustum = 28.49 L = $28.49 \times 1000\ cm^3$ = $28490\ cm^3$

Now, Volume of frustum = $\frac{\pi h}{3}\left(R^2 + Rr + r^2\right)$

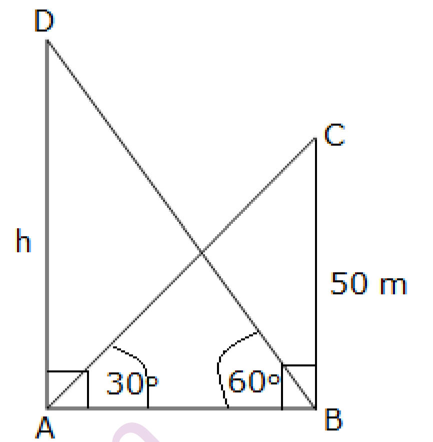
$$\Rightarrow \frac{22h}{7 \times 3}\left(28^2 + 28 \times 21 + 21^2\right) = 28490$$

$$\Rightarrow \frac{22}{21} h \times 1813 = 28490$$

$$\Rightarrow h = \frac{28490 \times 21}{22 \times 1813} = 15 \text{ cm}$$

Hence the height of bucket is 15 cm.

30. Let the height of hill is h.



In right triangle ABC,

$$\frac{50}{AB} = \tan 30° \Rightarrow \frac{50}{AB} = \frac{1}{\sqrt{3}} \Rightarrow AB = 50\sqrt{3}$$

In right triangle ABD,

$$\frac{h}{AB} = \tan 60° \Rightarrow \frac{h}{AB} = \sqrt{3} \Rightarrow h = \sqrt{3}AB$$

$$\Rightarrow h = \sqrt{3}\left(50\sqrt{3}\right) = 150 \text{ m}$$

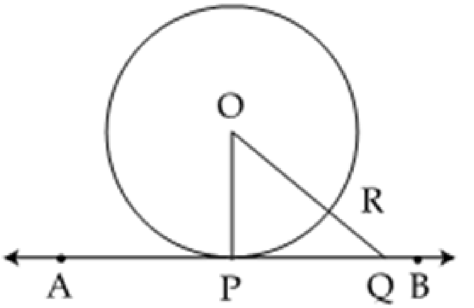
Hence the height of hill is 150 m.

31. Given: AB is a tangent to a circle with centre O.

To prove: OP is perpendicular to AB.

Construction: Take a point Q on AB and join OQ.

Proof: Since Q is a point on the tangent AB, other than the point of contact P, so Q will be outside the circle.



Let OQ intersect the circle at R.

Now OQ = OR + RQ

$\Rightarrow$ OQ > OR $\Rightarrow$ OQ > OP [as OR = OP]

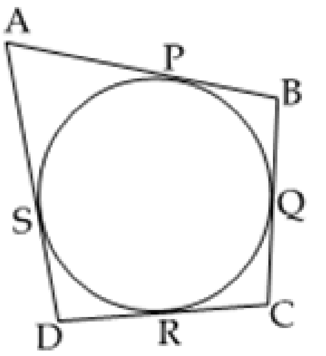
$\Rightarrow$ OP < OQ

Thus OP is shorter than any other segment among all and the shortest length is the perpendicular from O on AB.

$\therefore$ OP $\perp$ AB. Hence proved.

www.aklaninstitute.com

OR



Let ABCD be a quadrilateral, circumscribing a circle.

Since the tangents drawn to the circle from an external point are equal, we have

AP = AS ... (1)

PB = BQ ... (2)

RC = QC ... (3)

DR = DS ... (4)

Adding, (1), (2), (3) and (4), we get

AP + PB + RC + DR = AS + BQ + QC + DS

(AP + PB) + (DR + RC) = (AS + SD) + (BQ + QC)

AB + CD = AD + BC.

32. Total cost of books = Rs 80

Let the number of books = x

So the cost of each book = Rs $\frac{80}{x}$

Cost of each book if he buy 4 more book = Rs $\frac{80}{x+4}$

As per given in question:

$$\frac{80}{x} - \frac{80}{x+4} = 1$$
$$\Rightarrow \frac{80x + 320 - 80x}{x(x+4)} = 1$$
$$\Rightarrow \frac{320}{x^2 + 4x} = 1$$
$$\Rightarrow x^2 + 4x - 320 = 0$$
$$\Rightarrow (x+20)(x-16) = 0$$
$$\Rightarrow x = -20, 16$$

Since number of books cannot be negative,

So the number of books he brought is 16.

www.akilaninstitute.com

OR

Let the first number be x then the second number be 9 – x as the sum of both numbers is 9.

Now the sum of their reciprocal is $\frac{1}{2}$, therefore

$$\frac{1}{x}+\frac{1}{9-x}=\frac{1}{2}$$
$$\Rightarrow \frac{9-x+x}{x(9-x)}=\frac{1}{2}$$
$$\Rightarrow \frac{9}{9x-x^2}=\frac{1}{2}$$
$$\Rightarrow 18=9x-x^2$$
$$\Rightarrow x^2-9x+18=0$$
$$\Rightarrow (x-6)(x-3)=0$$
$$\Rightarrow x=6,3$$

If x = 6 then other number is 3.

And, if x = 3 then other number is 6.

Hence numbers are 3 and 6.

33. Given: $S_{20}=-240$ and $a=7$

Consider, $S_{20}=-240$

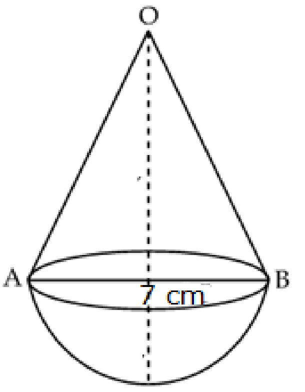
$$\Rightarrow \frac{20}{2}(2\times 7+19d)=-240 \qquad \left[\because S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right]$$
$$\Rightarrow 10(14+19d)=-240$$
$$\Rightarrow 14+19d=-24$$
$$\Rightarrow 19d=-38$$
$$\Rightarrow d=-2$$

Now, $a_{24}=a+23d=7+23\times -2=-39$

Hence, $a_{24}=-39$

www.akilaninstitute.com

34. Radius of hemi-sphere = 7 cm



Radius of cone = 7 cm

Height of cone = diameter = 14 cm

Volume of solid = Volume of cone + Volume of Hemi-sphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi r^2 (h + 2r)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 49(14 + 14)$$
$$= \frac{1}{3} \times \frac{22}{7} \times 49 \times 28$$
$$= \frac{22 \times 7 \times 28}{3} = \frac{4312}{3}\ cm^3$$

Radius of cylinder = Radius of cone = r = 6 cm

Height of the cylinder = Height of the cone = h = 8 cm

Slant height of the cone= $l = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ cm

Total surface area of the remaining solid

= Curved Surface Area of the Cylinder + Area of the Base of the Cylinder + Curved Surface Area of the Cone

$$(2\pi rh + \pi r^2 + \pi rl)$$
$$= 2x\frac{22}{7}x6x7 + \frac{22}{7}x6^2 + \frac{22}{7}x6x\sqrt{85}$$
$$= 264 + \frac{792}{7} + \frac{132}{7}\sqrt{85}$$
$$= 377.1 + \frac{132}{7}\sqrt{85} cm^2$$

www.akilaninstitute.com